IDENTIFICATION OF THE PARAMETERS IN EQUIVALENT THERMAL CIRCUITS OF ELECTRIC MOTORS IN THE NONSTEADY REGIME

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We have used the solution of the inverse problem to determine the causal characteristics of the heat-transfer process in electric motors.

Investigation and experimentation involving thermal processes play a significant role in the design of electric motors and in the selection of their regime parameters. Day-to-day experience shows that the success of such investigations depends to a significant extent on the reliability with which the experimental information is processed, as well as on the validity of the mathematical heat-transfer models that have been selected. An effective method for calculating the thermal state of electric motors as technical objects of complex construction is the method of equivalent thermal circuits (ETC) [1-5] according to which the object being investigated is conditionally divided into isothermal units which exchange heat with each other and with the ambient medium. It was demonstrated in [6] that when this division is sufficiently fine the ETC method is comparable in accuracy to the method of finite differences. Here we will also find a description of the method of directing the finite-difference schemes in the ETC to the steady-state regime. It is usually assumed [7, 8] that the utilization of the thermal conductivities calculated for the steady-state regime introduce no significant errors into the analysis of the nonsteady regime. As a matter of fact, we must treat this contention rather carefully if the object contains sources of heat that are subject to rapid changes in power; however, the division into minute details must be accomplished with acceptable accuracy.

When we use the ETC method to calculate the thermal state of objects, the mathematical model of heat transfer is a system of ordinary differential first-order equations

$$C_i \frac{dT_i}{d\tau} = \sum_{j \neq i} \lambda_{ij} (T_j - T_i) + P_i, \ i = \overline{1, N}, \quad (1)$$

$$T_i(0) = T_{0i}, \ i = \overline{1, N}, \quad (2)$$

where the parameters C_i, λ_{ij}, P_i in the general case are temperature functions of the ETC nodes, as well as functions of time.

The coefficients λ_{ij} in the rotated form include conductivities due to radiation, free and forced convection, thermal conductivity, contact conductivities through threaded and glued joints, and pressure fittings. These sources of heat exhibit diverse physical characteristics: Joule heat, friction in the gas clearances, and in the supports.

Such a multiplicity of physical heat-transfer phenomena in electric motors, some of which have by no means been studied adequately, leads to a situation in which a portion of the coefficients in the equations of system (1), (2) is known with a considerable level of indeterminacy in virtually any of the calculations.

Thus, the thermal regime of a stator, one of the most heavily loaded parts of a motor, is determined to a significant degree by the conductivity of the windings around the bundle of wires. The problem of finding the equivalent thermal conductivity λ_{eq} of the windings has been studied in numerous references, for example, in [9-12]; however, as indicated in [12], the scattering in the values of λ_{eq}, obtained by various methods, amounts to 30%.

As a consequence of the complexity of the geometry and inadequacy of information regarding a number of parameters, an analogous situation exists in the analysis of the radiation

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processes [1, 3, 13-16], as well as in the processes of free and forced convection [3, 17-21], and the heat generated in the gas clearances [3, 20, 21].

Thus, an investigation into the thermal state of electric motors in the nonsteady regime must make provision, as a necessary stage, for the correction of the mathematical model so as to identify the coefficients of system (1), (2) that have not been determined.

From the standpoint of cause and effect, the problem of identifying the ETC parameters is an inverse heat-exchange problem (IHEP) [4], for which the initial data, in addition to the structure of the mathematical model (1), (2), and the known coefficients C_i, λ_{ij}, P_i, are also represented by the temperatures of certain experimentally derived sections of the object.

Let us examine the extreme formulation of the IHEP, which exhibits a number of advantages over other formulations: the possibility of taking into consideration a priori information and "overdeterminacy," as well as utilization of the iteration regularity principle [4, 22-24].

Let the thermal state of the object be described by system (1), (2), for which the temperatures of certain units are known:

$$T_{i_k}(\tau) = T^*_k(\tau),\ k = \overline{1, K}, \qquad (3)$$

where i_k is the number of the section in which the k-th temperature sensor is located; K is the number of temperature sensors.

We have to determine the unknown coefficients of system (1), (2) from the condition

$$J = \sum_{k=1}^{K} \int_0^{\tau_m} [T_{i_k}(\tau) - T^*_k(\tau)]^2 d\tau \simeq \delta^2, \qquad (4)$$

where δ^2 is the total error in the temperature measurements and calculations.

We will assume that the identified coefficients of system (1), (2) are dependent on time. However, we can undertake the physical description of these characteristics after they have been determined to be functions of time. Thus, if it is the radiative conductivity between the i-th and j-th sections of the ETC that is being identified, i.e., $\lambda_{rad}(\alpha, T_i, T_j) = \alpha(T_i + T_j)(T_i^2 + T_j^2)$, and if the time-related distribution of this parameter is described by the function $\lambda^*(\tau)$, the identification of the radiative conductivity as a physical relationship reduces to the determination of the quantity α from the condition

$$\| \alpha [T_i(\tau) + T_j(\tau)] [T_i^2(\tau) + T_j^2(\tau)] - \lambda^*(\tau) \| \to \min_{\alpha}, \qquad (5)$$

where the norm $\|\cdot\|$ is given in the appropriate space, for example, in the functional space quadratically integrated on segment $[0, \tau_m]$.

In order to solve the IHEP by the gradient method, an effective approach involves the utilization of the problem from [25] conjugate to (1), (2). The structure of this conjugate system of ordinary linear differential equations with respect to the functions $\{\psi_i(\tau),\ i = \overline{1, N}\}$ will be determined by the structure of system (1), (2) and by the dependence of its coefficients on temperature and time:

$$-C_i \frac{d\psi_i}{d\tau} = \sum_{j \neq i} \lambda'_{ij} (\psi_j - \psi_i) + (C'_i + P'_i)\psi_i + 2\sum_{k=1}^{K} \delta_{i i_k} (T_{i_k} - T^*_k),\ i = \overline{1, N}, \qquad (6)$$

$$\psi_i(\tau_m) = 0,\ i = \overline{1, N}. \qquad (7)$$

Here $\lambda_{ij}' = \lambda_{ij} - (T_j - T_i)d\lambda_{ij}/dT_i$, if the conductivity λ_{ij} depends explicitly on the temperatures of the i-th and j-th sections, $\lambda_{ij}' = \lambda_{ij}$, if the conductivity λ_{ij} depends explicitly only on the time, the quantity $C_i' = dC_i/d\tau$ is different from zero only in the event of explicit relationship between heat capacity C_i and time, the quantity $P_i' = dP_i/dT_i$ is different from zero only when the generated heat P_i depends explicitly on the temperature of the i-th section.

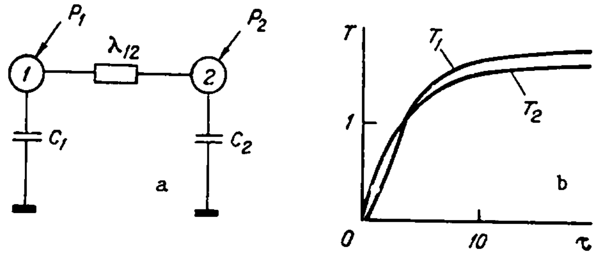

Fig. 1. Structure of the test ETC (a) and results from the solution of the direct problem (b).

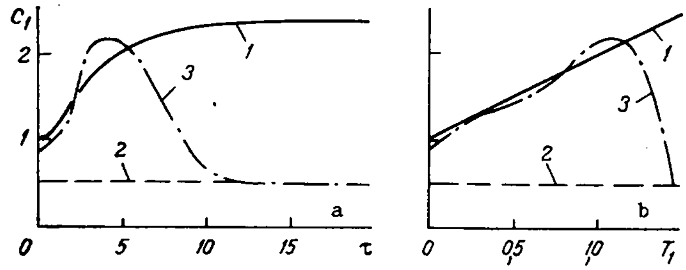

Fig. 2. Determination of the parameter C_1 as a function of time (a) and temperature (b): 1) actual solutions; 2) initial approximation; 3) determination of solution (20th iteration).

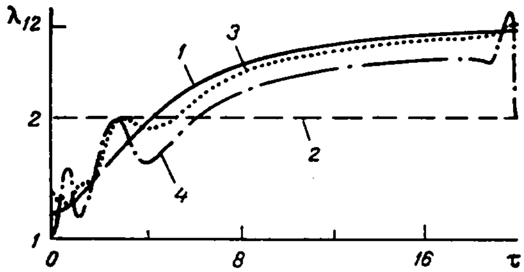

Fig. 3. Determination of the parameter λ_{12} over time as a function of the location of the temperature sensor: 1) actual solution; 2) initial approximation; 3, 4) determined solution with the sensor positioned, respectively, at the first and second nodes (20th iteration).

The components of the functional gradient are expressed in terms of the solution for the conjugate system in the following manner:

$$J'_{C_i}(\tau) = -\psi_i \frac{dT_i}{d\tau}, \tag{8}$$

$$J'_{\lambda_{ij}}(\tau) = \psi_i (T_i - T_j) + \psi_j (T_j - T_i), \tag{9}$$

$$J'_{P_i}(\tau) = \psi_i. \tag{10}$$

One method of solving the IHEP was to use the method of steepest descent with linear estimates of the optimum descent depths β_C, β_λ, β_P in each iteration.

To check on the operational validity of the algorithm, we introduced into our examination an abstract object modeled by a two-section ETC (Fig. 1a), with the following selection of parameter relationships: $C_1 = 1 + T_1$, $C_2 = 1 + 0.5T_2$, $\lambda_{12} = 1 + a(T_1 + T_2)^2$ for $a = 0.25$, $P_1 = T_1 \exp(-T_1)$, $P_2 = 1 - T_2$. The initial temperatures of the node sections, as well as the

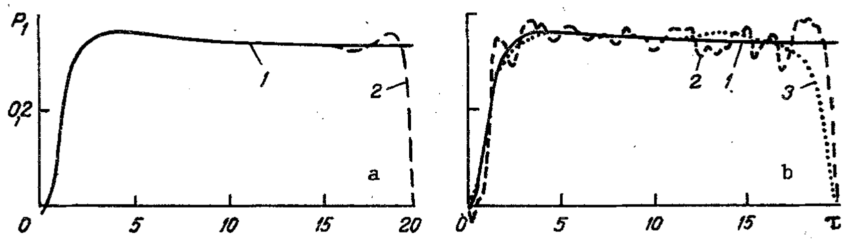

Fig. 4. Determination of the parameter P_1 as a function of time for unperturbed (a) and perturbed (b) temperatures on the basis of the normal law with $\sigma_T = 2\%\ T_{1max}$ (with an initial zeroth approximation): 1) real solution; 2) determined solution (20th iteration); 3) stop because of discrepancy (5th iteration).

temperature of the ambient medium, were assumed to be zero, while the inlet temperatures were those section temperatures calculated from the direct problem (Fig. 1b). The number of time intervals was set equal to 100 for $\tau_m = 20$.

Three IHEP were solved independently of each other in terms of the parameters C_1, λ_{12}, and P_1. The iteration process was stopped with the "joining" of the approximations for the unperturbed inlet temperatures and on the basis of the criterion of discrepancy for the inlet temperatures perturbed in accordance with the normal law.

The results shown in Figs. 2-4 allow us to draw the following conclusions. The quality of identification is determined by the location of the temperature sensors. The preliminary analysis of the sensitivity of the section temperatures to changes in the parameter λ_{12} demonstrated that the effect of this parameter on the temperature of the first section is considerably stronger than on the temperature of the second section, so that the result shown in Fig. 3 is completely explainable on the basis of the approach proposed in [26].

However, the method of positioning the temperature sensors is not the only factor which governs the accuracy with which the ETC parameters are identified. An important condition for the reduction in the error of determination is the selection of an appropriate thermal-regime control for the object in the experiment. For example, we can see from Fig. 3 that in the vicinity of the time point $\tau = 3$, regardless of the locations of the temperature sensors, the parameter λ_{12} cannot be determined, i.e., there is no uniqueness. It might be noted that at the same instant of time we have $T_1 = T_2$ (see Fig. 1b). Since the conductivity λ_{12} characterizes the temperature difference between the first and second sections, it becomes clear why the determination of the conductivity λ_{12} is impaired in the vicinity of this time point. The analogously poor determinability of the heat capacity C_1 over a significant segment of the time interval occurs for the same reason as on approximation to the steady thermal state which is a consequence of the corresponding choice of heat sources, the rate of change in section temperature drops sharply and, as a result, the influence of the heat capacity on the thermal state of the object diminishes significantly.

Thus, the selection of the control mechanism during the course of the experiment to measure the temperatures at these sections cannot be arbitrary. Excellent reproducibility of heat capacity for any section of the ETC in a given time or temperature interval presupposes the nonsteadiness of the thermal regime of this section in this interval. Precisely in the same way, the greater the temperature differences between T_i and T_j, the better the reproducibility of the conductivity λ_{ij} that we can expect.

As is demonstrated by the results of the calculations, reproducibility of the parameters is absent in each and every case when $\tau = \tau_m$. It was indicated in [4] that the accuracy of the calculation can be increased by expanding the time interval, through utilization of the a priori information regarding the parameter to be identified in the choice of the initial approximation and in taking smoothness into consideration.

All of the above-enumerated unique features involved in the reproducibility of the parameters in system (1), (2) are closely associated with the problem of uniqueness in the solution of the IHEP. The formulation of the inverse problem in which the quantities to be identified are regarded to be functions of time assumes a rather broad class of permissible solutions. Under these conditions, the uniqueness theorems [27-30] play a principal role, and these impose certain conditions on the IHEP data, of which we actually spoke earlier. However, it is assumed in the cited references that the heat-transfer process follows the Fourier law (distributed model) in canonical bodies. At the same time, rigorous conditions of uniqueness for the solution of the IHEP, formulated for model (1), (2), are extremely urgent, since the search for the unknown parameters of the ETC as functions of time is governed by the fact that in the presence of complex geometric shapes in the nonsteady case there is an absence of sufficiently simple and reliable physical relationships linking these quantities, such as, for example, the conductivities of free and forced convection, contact conductivities, etc. This inadequate theoretical research base with respect to a number of phenomena relating to the transfer of heat in electric motors compels us to resort to the identification of ETC parameters in the form of functions of time.

It should be noted that as we turn from the identified parameters to their physical relationships rather smoothly the validity of the inverse problem is improved, since, as was noted in [4], in this case the class of permissible solutions is narrowed. Thus, determination of the coefficient for the nonlinear component in the conductivity λ_{12} from the condition (5) in the presence of perturbations in the inlet temperature ($\sigma_T = 3\% \; T_{1max}$) yields a value of $a = 0.235$, which is sufficiently close to the actual value $a = 0.25$.

NOTATION

T_i, temperature of the i-th node; C_i, heat capacity of the i-th node; λ_{ij}, thermal conductivity between the i-th and j-th nodes; P_i, power of the heat generated in the i-th node; τ, instantaneous time; J, specific functional; ψ_i, conjugate function; σ_T, mean-square deviation in the normal perturbation law; a, coefficient of nonlinear identified conductivity in the test problem.

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INVESTIGATING THE DIAGNOSTICS ALGORITHMS OF THE THERMAL EFFECT IN DESIGN

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We examine the iteration solution algorithms of inverse heat-conduction problems (IHCP) with consideration of some a priori information regarding the sought relationship.

In recent times, the derivation of the characteristics applicable to heat-exchange processes based on the methodology of inverse problems has gained increasing applicability, both in the processing of experimental results, as well as in the construction of mathematical models of real processes. This has stimulated further development of solution algorithms for inverse heat-exchange problems and more extensive investigation of their properties from the standpoint of practical application.

Let us formulate some inverse problems in the form

$$Au = f,\ u \in U,\ f \in F, \tag{1}$$

where A: U → F is a nonlinear operator in the general sense; U and F are Hilbert spaces. We know from physical considerations that u, as a rule, is a smooth function. Therefore, for U we make use of the Sobolev space $W_2{}^k$. The function f, since it is a result of measurements, is generally known with some error and represents a rather arbitrary relationship f_δ. Naturally in this case the space L_2 must be examined from the standpoint of F.

The operator that is the reciprocal of A is usually bounded, i.e., the formulated problem is incorrect and for its solution we must make use of regularizing algorithms. In the following, for this purpose, we employ a method based on iteration regularization. Research has shown [1] that excellent effectiveness is achieved by IHCP solution algorithms based on a scheme from the method of conjugate gradients, where the iteration number k is taken as the regularization parameter

$$u_{k+1} = u_k - \beta_k p_k,\ k = 0,\ 1,\ \ldots,\ K^*, \tag{2}$$

where the direction of descent

$$p_{k+1} = J'_{W_2{}^k}(u_{k+1}) + \gamma_{k+1} p_k, \tag{3}$$

$$\gamma_0 = 0;\ \gamma_{k+1} = -\frac{(J'_{W_2{}^k}(u_{k+1}),\ J'_{W_2{}^k}(u_k) - J'_{W_2{}^k}(u_{k+1}))_{W_2{}^k}}{\| J'_{W_2{}^k}(u_k)\|^2_{W_2{}^k}};$$

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